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APPENDIX B: MTE219 PROJECT REPORT EVALUATION

Category	Criteria	Grade
Format, Appearance, Writing Style	Clarity of communication Spelling, grammar and punctuation Organization and structure Neatness and completeness Title page, table of contents, lists of figures and tables Print quality and binding	9-10 Excellent 7-8 Good 5-6 Marginal 0-4 Unsatisfactory **10** /10
Executive summary	Purpose Methodology Results	3 Excellent 2 Good Marginal 1 Missing 0 **3** /3
Design: Description	Review of the design problem Design constraints and criteria Materials and properties: report your lab results (data and analysis) here *-2 (Compare Lab Results with provided properties)*	15-17 Excellent 12-14 Good 8-11 Marginal 0-7 Unsatisfactory **15** /17
Design: Synthesis	Three conceptually different designs Free hand sketches and features review Concepts evaluations Design selection *-2 (Quantitative selection criteria)*	13-15 Excellent 10-13 Good 7-10 Marginal 0-7 Unsatisfactory **13** /15
Design: Analysis	Completeness of the analysis *+2 (Explicit soultion of member geometry as a function of applied load)* Adequacy and suitability of analysis Accurate and free of errors analysis; Analysis accuracy validation *+1 (Material Anisotropy)* *+1 (Torsional Stability optimization)* Design optimization *+1 (Stress concentration optimization)* Drawings of the final design	27-30 Excellent 21-26 Good 15-20 Marginal 0-14 Unsatisfactory **35** /30
Design: Construction & Testing	Test results and analysis Design refinements	9-10 Excellent 7-8 Good 5-6 Marginal 0-4 Unsatisfactory **10** /10
Project Overall	Creativity *+3 (Design 3 Creativity)* Lab activity (analysis and presentation) Organization and construction execution	13-15 Excellent 10-13 Good 7-10 Marginal 0-7 Unsatisfactory **18** /15

General Comments: It is evident the amount of work that was put into your report which excellently documented the design and analysis process. Your observations on the effects of stress concentrations, material anisotropy, torsional stability etc. are by far the most comprehensive explanations which were presented that explain the differences between the analytical prediction and the actual result. Your real world application section also displays the mark of thinking like a true engineer. Your explicit solution of the problem analytically was very well laid out and made your design iteration and subsequent calculations very quick as you converged to a further optimized physical result with a better correlating analytical model. Excellent job, all of your group should be very proud of this great achievement.

Total 104 /100

UNIVERSITY OF
WATERLOO

Department of Mechanical & Mechatronics Engineering
MTE 219 Mechanics of Deformable Solids
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Optimum Design of a Crane Boom Truss

Concepts, Design Analysis, Construction, Optimization, and Performance

Prepared by **Group 28:**

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1. Executive Summary

1.1. Purpose

With the rate in which the construction industry is developing, there is a demand for improved technology to better facilitate the engineering and construction process. With any engineering design project, several factors need to be carefully calculated and considered in order to meet the given constraints and criteria. The purpose of this project was to design and build a truss using balsa wood. The goal was to get the highest performance value possible, determined by dividing the mass the truss held by the mass of the truss itself. This was to be done in accordance with the given set of constraints, such as the length, width, and height limitations, along with the amount of material provided. The material properties of the balsa wood and hardwood dowels to be used were either given or found through laboratory experiments.

1.2 Methodology

The methodology employed to design the truss could be broken down into three phases, the truss design, member optimization, and test iterations. First, the optimal truss was determined by taking into account factors such as number of members and force distribution. After various concepts, a truss was decided upon. A force analysis was done on the final truss to determine the tensile and compressive forces in each member. Since the mass of the truss is inversely related to the performance value, it was beneficial to remove any excess weight on the truss. The best way to do that was to design the truss in such a way that failure in all the members occurred at the same time. Failure mode calculations were done for every failure mode, and equations were found relating member dimensions to the force applied at the end of the truss. After considering each failure mode, a limiting force was determined and substituted back through all of the other equations, thus yielding the dimensions of the truss. Unfortunately, it was later discovered during the experimentation phase that while the mathematical approach to achieve optimal member construction provided a strong starting point, the performance in the real world was far less superior to what the math proved. This was possibly due to failure modes not being considered, as well as other oversights in member designs such as stress concentrations. In conjunction with the initial mathematical analysis, careful planning and efficient use of available material allowed for five design iterations (including the final truss). Each of these design iterations was tested to failure, and improvements were made on subsequent iterations based on the results, until the final truss was ready.

1.3 Results

The final demonstration revealed that the truss designed had the highest performance value of the entire class. The final truss iteration weighed 16.08 grams and was able to hold a mass of 5.0kg, yielding a performance value of approximately 311. This proved to be a great success as the major constraint to reach a performance value of 125 was far surpassed. This project was integral in bringing to light the careful considerations and arduous attention to detail that is required in any design project, whether in the walls of an academic institution, or the real world. The calculations predicted that a truss half the weight of the final one, would hold a weight twice that of the final truss. The calculations and the reasoning for these discrepancies have been discussed in length in the sections below.

2. Introduction

2.1. Problem Review

The problem, as illustrated in Figure 1, dictates that a 30 cm long crane boom truss is to be designed with balsa wood such that it has the highest performance value possible. The boom is to be loaded at the free end, 30 cm from a 5x6 cm base that contacts the wall. The top end of the truss is supported by a pair of hooks placed roughly 6 cm apart. These will serve as pin supports while the bottom ends will rest against the wall, serving as roller supports. The truss is meant to have a minimum performance value of 75. For a mass to qualify as being supported by the truss, it must be supported for more than five seconds.

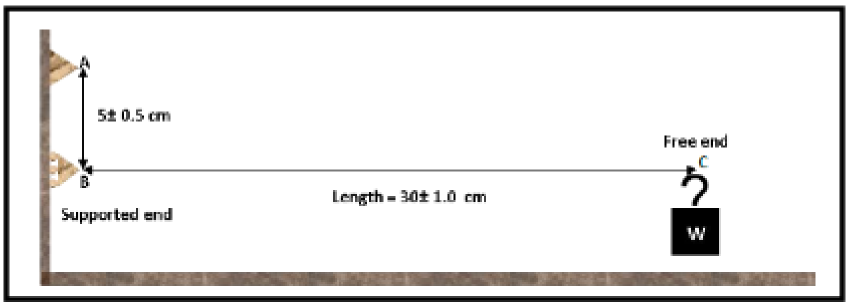

Figure 1. Project problem schematic description

2.2. Constraints

There are several constraints that needed to be considered when designing the crane boom. First, it was to be a truss design consisting of only two force members connected by pin joints. These pins were to be free to rotate such that the members cannot be stuck to the pins. The truss was to have a length of 30 ± 1 cm, and the end connecting to the wall was to have a height of 5 ± 0.5 cm along with a width of 6 ± 0.5 cm. The load was to be added only to the dowel at the free end of the truss. Attaining a performance value of 125 was also set as an important constraint to be considered while designing the truss. The amount of material available was restricted to an area of 324 in^2 of balsa wood having a thickness of 3.25mm.

2.3. Criteria

The most important criteria to consider when designing this truss was to keep the mass low. Mass of the truss is inversely related to performance value, thus reducing mass can help to increase performance value. Another criteria was to try and reduce the effect of compression members in the design as balsa wood has a much higher tensile strength than compressive strength. It was desired to design each individual member in such a way that they would all fail at the same time. This could be used as a basis for calculating the dimensions necessary for the members so that unwanted material did not add to the weight of the entire truss assembly.

2.4. Material Properties

Over the course of the initial phase of the truss design, two tests were conducted to determine the material properties of balsa wood. The first one was a three point bending test to determine material properties such as buckling and ultimate tensile strength. The second one was the cantilever beam test which was used to find the ultimate shear strength and Elastic modulus. The following results were obtained for the balsa wood and the hardwood to be used for the dowels.

Table 1: Material Properties of Balsa Wood

Density	0.141 g/cm ± 0.03
Ultimate Strength	14.6 Mpa ± 5.8
Shear Strength	2.05 Mpa ± 0.36
Elastic Modulus	3.6 GPa ± 0.8

Table 2: Material Properties of Hardwood Dowels

Density	650 g/m
Elastic Modulus	17 GPa
Normal Strength	117 MPa
Maximum Bending Moment	368 N.mm
Shear Strength	23 MPa

3. Preliminary Designs

3.1. Concept Generation

The first part of the design process was deciding on which truss configuration to use. The overall truss design was very important as it would provide the fundamental framework when further optimizing and designing the members. Brainstorming different truss designs was done using Autodesk ForceEffect, a conceptual tool that provides calculations of forces in free body diagrams, including static structures. This allowed a large number of designs to be analyzed quickly, allowing more ideas to be tested and increasing the chance of selecting the best truss. Needless to say, several factors needed to be considered even in the early phases of the planning; the magnitude of factors to consider made it clear that it was not necessarily obvious as far as which design was best. It was determined however, that as long as most of these factors were carefully considered, any shortcomings in the overall designs could be salvaged through individual member designs (for example, if there were members with high forces, they could easily be strengthened to decrease their stress, etc.). These considerations included creating a truss with a low mass, with minimal maximum forces and low compressive forces in all members.

One criteria was to design trusses with low masses. Since the main goal of this design was to hold 125 times the weight of the truss, a lighter truss would make it easier to meet this goal. Theoretically, a heavier truss should be stronger, meaning it could also hold 125 times its weight; however, this mass constraint was mainly set to avoid unnecessary and over-designed members in the preliminary concepts. The next desirable criteria was minimizing the maximum forces in the truss. Having certain members with very high forces would extra fortification and larger cross-sectional areas, which along with glue, could add substantial weight to the design. Lower maximum forces were also desired because of pin shear. Pin shear depends on the force and the cross sectional area of the pins, and the only way to change the cross sectional area of the pins is to introduce more than one dowel. This adds complexity to cutting the pin joint hole, and adds complications with regards to the other failure modes, especially with plate shear. Double shear could be introduced - however, that adds more weight. Hence, the best way to reduce the shear stress in the pins is to reduce the force in the member. The pins will fail at a certain stress based on the shear strength material property. Since there is not much that can be done as far as member design, this relies heavily on the truss design and specifically, lowering the maximum force in any one member of the truss. Finally, it was desirable to find a truss that had a low maximum compression members. Balsa wood is very weak in compression due to buckling. If two members that had the same stress were compared, but one was in compression while the other was in tension, the compression member would fail first. Since balsa wood is much weaker in compression than in tension, compression members with high forces would need to be strengthened. This would add a lot of extra weight to the truss as a whole, and would introduce other problems relating to the design of the compression members. For example, increasing cross sectional area is not sufficient to avoid buckling; avoiding buckling involves increasing the second moment of area which results in larger, bulkier members (I-beams, box beams) that can be more difficult to incorporate into the truss. Obviously this all comes down to member design (as bigger bulkier members may end up working better if designed correctly), but it was important to note that there was no need for over-designed, unnecessarily strong members that would merely add weight; the goal was to instead create a truss that would facilitate member design and incorporation later on. All in all, these aforementioned factors did not guarantee that the truss design itself would be the most efficient because that would stem primarily from member design. However, attempting to meet these criteria would, in theory, create a strong foundation for the next steps of member design and optimization. Picking an effective design in this phase would also make member design easier and would ideally result in a better performing truss.

3.1.1. Design One

The first concept was a simple triangular truss with three members. This concept, shown in Figure 2 below, is a low mass design. The compression and tension forces for the members provided a baseline of what these values would be for the simplest possible design. As a function of the force applied in newtons at the free end (point A in this case), member AB is in compression with a force of 6F, while member AC is in tension with a force of 6.083F. Member BC is a zero-force member. F is defined as the force applied to the end of the truss by the weight.

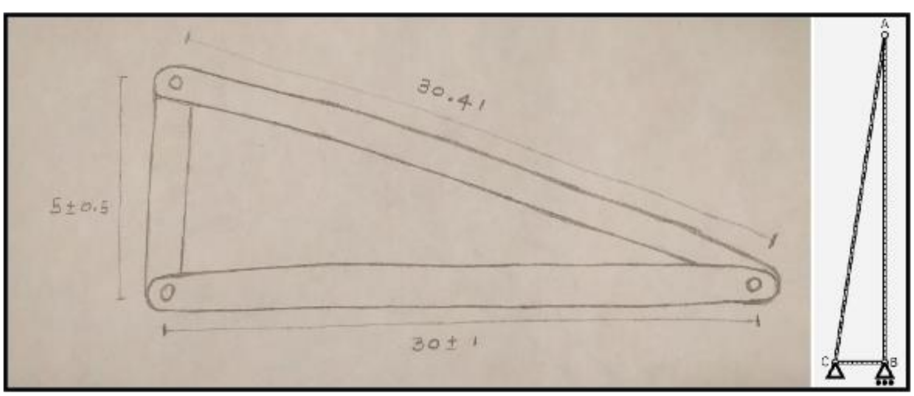

Figure 2. First design concept - a simple triangular truss

While this design had a low mass, the forces in the members were very high as compared to future design iterations (discussed below). As aforementioned, the high forces in the members would require further considerations for using multiple pins, along with fortifications to increase the cross-sectional areas (such as gluing multiple members together). This would in turn increase the mass of this low-mass design nonetheless. Moreover, the high compression member in this design presented a problem; due to the very low compressive strength of balsa wood when compared to its tensile strength, fortifying high compression members to avoid buckling would require the design of I-beams or box-beams, increasing the mass of the design once again.

3.1.2. Design Two

The second design which was considered is illustrated below in Figure 3. Each of the diagonal members DG, FI, and HA are in compression, with forces ranging from 1.56F to 1.72F. The members FG and HI are in tension, with a force of 1F. Member DE is a zero-force member. The diagonal members of the truss, as well as members FG and HI do well to distribute the forces evenly amongst them. Members BE, EG, GI, and IA are compressive members, their forces ranging from 2F to 4.6F. F is once again defined as the force applied to the end of the truss by the weight.

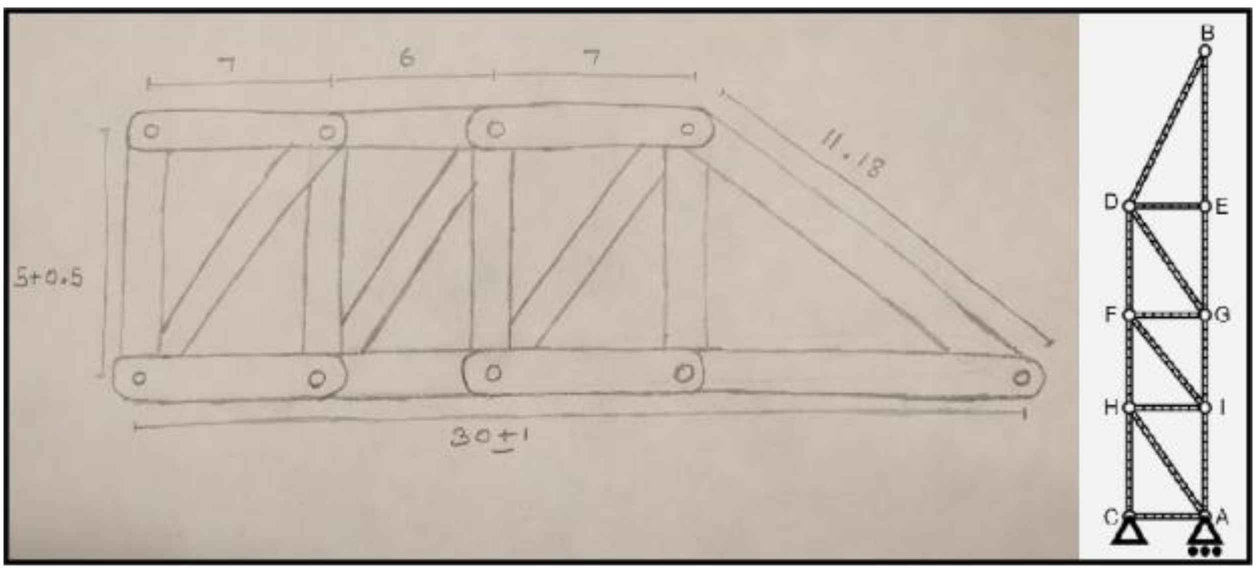

Figure 3. Second design concept

This design included several members as an attempt to distribute the force of the weight amongst all the members of the design. While using more members was effective in minimizing the maximum forces in each member, this caused the truss to weigh significantly more. Nevertheless, as compared to future design iterations, the maximum forces were still relatively high.

3.1.3. Design Three

The third design that was considered was a pentagonal-shaped truss with an ‘x’ shape focused in the middle. This was a mirrored space truss with 10 members on each side. This truss can be seen in Figure 4. This truss was interesting because it moved outside the theoretical (30 cm by 6cm) box that defined by the constraints. This departure actually provided a number of benefits. First, it was able to distribute the stresses effectively. The largest compressive stress in any member was 3.109 times the total force applied at the free end, which was very good, especially compared to other designs. This was also the maximum for any member, which was excellent for pin shear, because it meant that more force could be applied to the free end before pin shear became a factor (versus a design that had a higher maximum force in relation to applied load). The total number of members required for this truss was in the middle of the spectrum when compared with other designs. The idea with this truss however was that since the forces were distributed so well, there would not have to have as much fortification with these members since they would not individually carry forces quite as large.

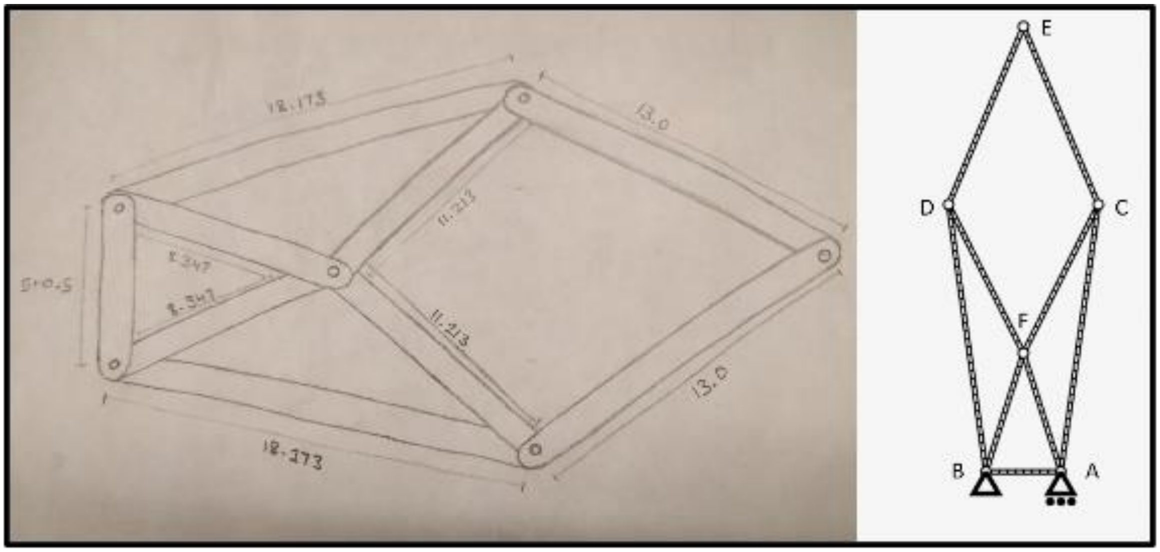

Figure 4. Third design concept

3.2. Design Selection

In the end, the third design (pictured above in Figure 4) was selected as the truss to construct. The main reasoning behind this choice was the fact that the forces were distributed so well. Shortcomings in other designs could have been overcome through effective member design and optimization; however, this design ensured a strong foundation where less member design and optimization had to be done to achieve the same level of success. As well, one thing that would be much more difficult to design was pin shear. As stated above, pin shear is difficult because the only way to really reduce the effects of pin shear is to make it double shear, add weight to the trusses, or add multiple pins. All of these options would require a lot of extra design or weight, so the desire was to avoid this as much as possible. This truss provided the best distribution, meaning that pin shear would not play as large of a role as it would have with the other designs. The main problem with design one was that it had extremely large forces in its members. The problem here was that the members would need to be fortified and made large such that they would add a lot of weight to the low mass design regardless. As well, pin shear would become a huge factor, and accommodations for this fact would most likely have resulted in more weight and tougher design decisions. The problem with the second design was that had a much higher mass and many more members. The intent of introducing more members in the design was to distribute the forces effectively between them, but this was not done as well as initially expected, especially when compared with the third design.

The selected truss had a zero force member above the ‘x’ in the frame (connecting C and D) in the initial design. This member was removed since it added nothing to the structure but weight. The final truss can be seen above in Figure 4 and below in Figure 5.

It is imperative to keep in mind that it was not guaranteed that the truss selected was the best truss to use. That is because any of the factors that were problematic for the other trusses could have been fixed with member optimization. However, this truss gave the best chance at an efficient and strong starting point with which to begin the optimization. It minimized tough design decisions and eliminated some of the larger potential failure modes.

4. Design Analysis and Optimization

4.1. Design Analysis

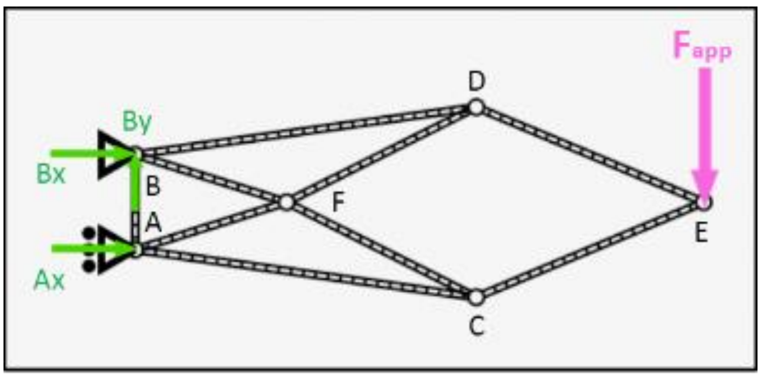

Figure 5. Final truss as a body

4.1.1. Two-Dimensional Force Analysis

For the purpose of calculating the force in each member, the method of joints was used. The force analysis for the entire truss as a rigid body, analysis of one joint, and the resultant forces of the remaining members are given below. The force in each member is written as a function of F_{app}, which is the force applied at the end of the truss as shown above.

Considering the entire truss as a rigid body will allow the support reactions to be solved.

$$A_x = -B_x \; since \; \Sigma F_x = 0$$

$$A_y = F_{app} \; since \; \Sigma F_y = 0$$

$$\Sigma M_A = 0$$

$$5B_x - 30F_{app} = 0$$

$$B_x = 6F_{app}$$

$$A_x = -6F_{app}$$

The calculations for one joint are displayed below. These calculations are done for joint E. Figure 6 provides an illustration of the joint being analyzed and the forces acting on it.

At Joint E,

$$F_{app} = F_{DE} sin\alpha + F_{CE} sin\beta \; since \; \Sigma F_y = 0 \; (1)$$
$$F_{CE} cos\beta = F_{DE} cos\alpha \; since \; \Sigma F_x = 0$$

Since it is known that $\alpha = \beta$(2)

$$F_{CE} = F_{DE} \; (3)$$

Solving (1), (2), and (3) gives:

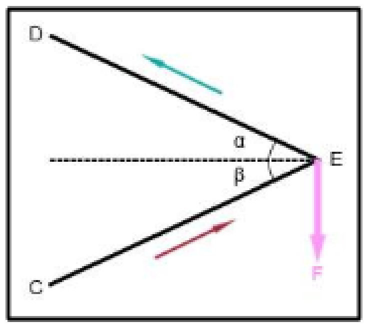

Figure 6. Method of Joints at Point E

$$F_{DE} = 1.300F_{app} \; (T)$$
$$F_{CE} = 1.300F_{app} \; (C)$$

The process was continued until the force in each member was found. The results are below. The detailed calculation can be found in Appendix A.

$F_{AB} = 0.511F_{app} \; (T)$
$F_{AC} = 3.109F_{app} \; (C)$
$F_{BD} = 3.109F_{app} \; (T)$
$F_{AF} = 3.061F_{app} \; (C)$
$F_{BF} = 3.061F_{app} \; (T)$
$F_{CF} = 2.082F_{app} \; (T)$
$F_{DF} = 2.082F_{app} \; (C)$

These values can now be used in the stress and failure mode analysis to design the members.

4.1.2 Stress and Failure Mode Analysis

The goal for the stress and failure mode analysis was to maximize each member by designing them to fail at the exact same time. For that reason, each failure mode was analyzed to find an equation relating cross sectional area and the original force applied to the end of the truss. That way, a force could be chosen, and then the cross-sectional areas of every member could be calculated, such that every member would fail at the same time. For all of the following calculations, F_{app} will be used to denote the force applied to the end of the truss. Keep in mind this is only half of the actual force that would be applied since these calculations only account for half of the truss. In each calculation, the force F is replaced with the multiple of F_{app} corresponding to the member used (determined in the 2D analysis in

Section 4.1.1). This way, each calculation will yield a relationship between the cross-sectional area or member dimensions and the force applied at the end of the truss. The max stress values were obtained from the material properties listed in Section 2.4. The remaining calculations can be found in Appendix B.

Plate rupture for tension members is found using the following equation, where σ is the max plate normal stress, F is the axial force applied to the member, and A is the cross sectional area. *A = tw*, where t is the thickness of the member and w is the width of the member. The following calculation is for member AB.

$$\sigma = \frac{F}{A}$$

$$14.6x10^6 = \frac{0.511F_{app}}{A}\ m^2$$

$$A = tw = \frac{0.511F_{app}}{14.6}\ mm^2$$

The same was done for the remaining tension members, yielding:

BD: $tw = \frac{3.109F_{app}}{14.6}\ mm^2$

CF: $tw = \frac{2.082F_{app}}{14.6}\ mm^2$

BF: $tw = \frac{3.061F_{app}}{14.6}\ mm^2$

DE: $tw = \frac{1.300F_{app}}{14.6}\ mm^2$

Plate shear for tension members can be found using the following equation, where τ is the max allowable plate shear stress, F equals axial force in the member, b is the length from the side of the pin hole to the end of the member (along the length of the member) and t is the thickness of the member. The following calculation is for member BD.

$$\tau = \frac{F}{2bt}$$

$$2.0x10^6 = \frac{3.109F_{app}}{2bt}\ m^2$$

$$bt = 0.7712F_{app}\ mm^2$$

The same was done for the remaining members, yielding:

AB: $bt = 0.1250F_{app}\ mm^2$

CF: $bt = 0.5183F_{app}\ mm^2$

BF: $bt = 0.7716F_{app}\ mm^2$

DE: $bt = 0.3250F_{app}\ mm^2$

Bearing stress can be found using the following equation, where σ is the minimum normal stress (plate versus pin, plate is minimal in this case), F is axial force in the member, t is thickness of the member, and d is diameter of the pin joint hole. The following calculation is for member AC.

$$\sigma = \frac{F}{td}$$

$$14.6x10^6 = \frac{3.109F_{app}}{td}\ m^2$$

$$td = \frac{3.109F_{app}}{14.6}\ mm^2$$

The same has been done for the remaining members, yielding:

AB: $td = \frac{0.511F_{app}}{14.6}\ mm^2$

CF: $td = \frac{2.082F_{app}}{14.6}\ mm^2$

DF: $td = \frac{2.082F_{app}}{14.6}\ mm^2$

BD: $td = \frac{3.109F_{app}}{14.6}\ mm^2$

DE: $td = \frac{1.300F_{app}}{14.6}\ mm^2$

CE: $td = \frac{1.300F_{app}}{14.6}\ mm^2$

BF: $td = \frac{3.061F_{app}}{14.6}\ mm^2$

AF: $td = \frac{3.061F_{app}}{14.6}\ mm^2$

Compressive buckling can be found using the following equation, where F is the compressive force in the member, E is the modulus of elasticity of the balsa wood, I is the second moment of area of the member, and l is the length of the member. The following calculation is for member AC.

$$F = \frac{4\pi^2 EI}{l^2}$$

$$3.109F_{app} = \frac{4\pi^2(3.66x10^9)I}{0.18173^2}$$

$$I = 0.64678F_{app}\ mm^4$$

The same has been done for the remaining compressive members, yielding:

AF: $I = 0.14881F_{app}\ mm^4$

CE: $I = 0.15205F_{app}\ mm^4$

DF: $I = 0.18039F_{app}\ mm^4$

I (second moment of area) can be calculated using the following formula, where b is the base of the cross sectional area and h is the height of the cross-sectional area. These change based on what axis it is taken in reference to, so to ensure that both axis have the same second moment, the cross section will be made to be a square, so h=b. This substitution gives:

$$I = bh^3 = b^4$$

AF: $b^4 = 0.14881F_{app}\ mm^4$

CE: $b^4 = 0.15205F_{app}\ mm^4$

DF: $b^4 = 0.18039F_{app}\ mm^4$

AC: $b^4 = 0.64678F_{app}\ mm^4$

Plate rupture in compression is unlikely because the member will most likely buckle first. However, it will still be taken into consideration for thoroughness. The formula is exactly the same as plate rupture in tension. The sample calculation is for member CE.

$$\sigma = \frac{F}{A}$$

$$7.0x10^6 = \frac{1.300F_{app}}{A} \ m^2$$

$$A = tw = \frac{1.300F_{app}}{7} \ mm^2$$

The same process has been done for the remaining members, yielding:

AF: $tw = \frac{3.061F_{app}}{7} \ mm^2$

AC: $tw = \frac{3.109F_{app}}{7} \ mm^2$

DF: $tw = \frac{2.082F_{app}}{7} \ mm^2$

Pin shear is calculated using a different process since it turns out to be the limiting factor for this truss design. Pin shear stress depends on force and pin cross-sectional area. As stated above, it is desired for this design to have a maximum of one dowel per pin joint. That means that cross-sectional area is constant for the dowel. In previous calculations, both the cross-sectional area and the force could vary. Since a relationship will be obtained between the cross sectional area and the force for pin shear, the cross-sectional area of the dowel can be substituted, and force can be solved. This is the limiting force, and can be substituted back through the rest of the equations, yielding cross-sectional areas and subsequently the needed dimensions which can be used in the design of the members. The calculations were done for this pin because the maximum pin shear stress is desired to find the limiting force. The

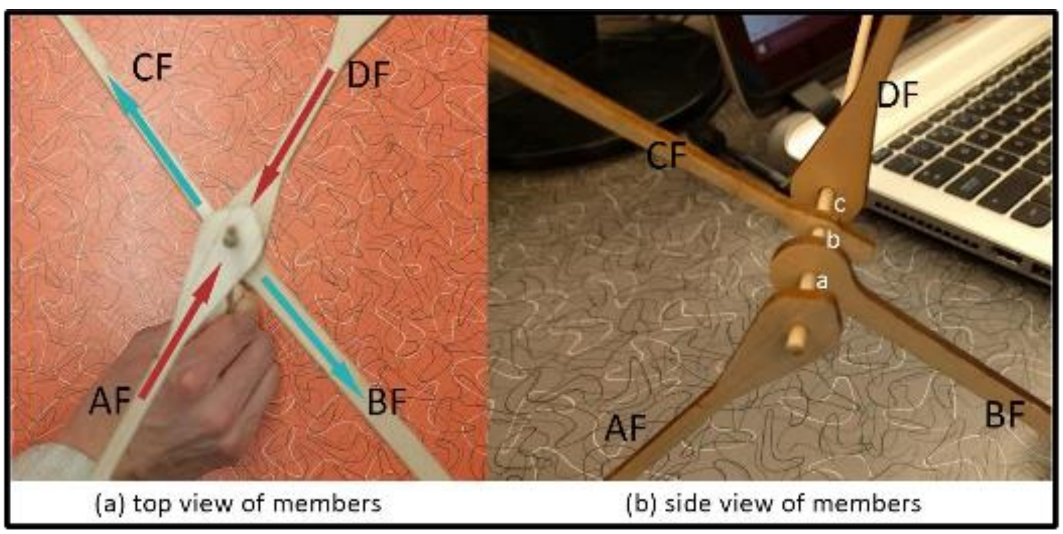

Figure 7. (a) A head-on view of the pin joint; (b) An exaggerated side view of the pin joint

maximum pin shear occurs here. Figure 7 will be used to outline the calculations.

Pin shear is different at a, b, and c. All three will be calculated and the largest stress will be used to calculate the limiting force. F_A, F_B and F_C occur at the corresponding points in Figure 7b. The calculation is for pin F.

$$F_A = F_{AF} = 3.061F_{app}$$

$$F_B = F_{AF} + F_{BF}$$

$$F_B = 3.061F_{app}sin8.71385i + 3.061F_{app}cos8.71385j + 3.061F_{app}sin8.71385i - 3.061F_{app}cos8.71385j$$

$$F_B = 6.1724F_{app}sin8.71385i$$

$$|F_B| = 0.76703F_{app}$$

$$F_C = F_{AF} + F_{BF} + F_{CF} = F_B + F_{CF}$$

$$F_C = 0.76703F_{app}i - 2.082F_{app}cos63.5172i + 2.082F_{app}sin63.5172j$$

$$F_C = -0.1574F_{app}i + 1.85547F_{app}j$$

$$|F_C| = 1.862F_{app}$$

Thus, the max stress occurs at point a. This point will then be used to solve for the limiting force using

$$\sigma_A = \frac{F}{A}$$

$$23x10^6 = \frac{3.061F_{app}}{0.25\pi(3.125)^2}$$

$$F_{app} = 57.16N$$

the known cross sectional area and max stress values.

Important Note:

Hence, the limiting force is 57.16N. That means that if a F_{app} of 57.16N is assumed, that value can easily be substituted into all of the above equations, yielding the corresponding cross-sectional areas and measurements for each member, as shown in detail in Appendix A. With these values, the CAD can be then be made to laser cut the first iteration. Since this is half the truss, the truss should fail at a load of 114.32N. Dividing by 9.81 yields the mass required to exert such a force, resulting in 11.65 kilograms. While this is theoretically the mass that can be held, it is unlikely to hold this, or even close to this, because the calculations assume ideal conditions, which unfortunately do not occur in the real world. As well, there may be other modes of failure that are unknown and therefore cannot be accounted for in these calculations. A safety factor of 2 can be applied, meaning that it should be able to hold 5.83 kilograms. A more conservative safety factor of 4 means the truss can hold about 2.91 kilograms, which is a much more reasonable estimate. Increasing the factor of safety, as mentioned, accounts of the chance of variability in the material properties of members. It also considers any non-linearities in the system (such as out of plate bending, friction in the joints, twisting, unexpected force concentrations at various points along members, along with any other unconsidered modes of failure). While all these calculations for cross-sectional areas and measurements for each member provide a good starting point, it became evident quickly that the mathematical analysis did not comply with the conditions of the real world. Detailed in Sections 4 and 5 are the steps taken to optimize and refine the design through various iterations, all the while relating the outcomes and reasons of unexpected failure to this mathematical analysis.

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Other things to be considered are the three point and four point bend in the dowels. The dowel at the end (joint E) where the load is applied experiences a three point bend, with the load in the middle pulling down and either side of the truss resisting that equally. Pin B, where the hooks attach, experiences four point bending, with the members on each side pulling out and the hooks in the middle pulling back towards the stand. These need to be considered to ensure the dowels do not break. If it is found that these are a concern, one way to remedy this could be to glue a second dowel to in between the members, attached to the original dowel. This would increase the cross sectional area at the center of the dowel, where it is most important.

Three point bending is calculated below.

The forces on this dowel are easy to calculate. The force pulling down is $2F_{app}$, because this is the total mass on the truss and F_{app} was previously defined as the force on half of the truss. The two supports on either side are F_{app}, since each side takes half. This can be seen in Figure 8 below.

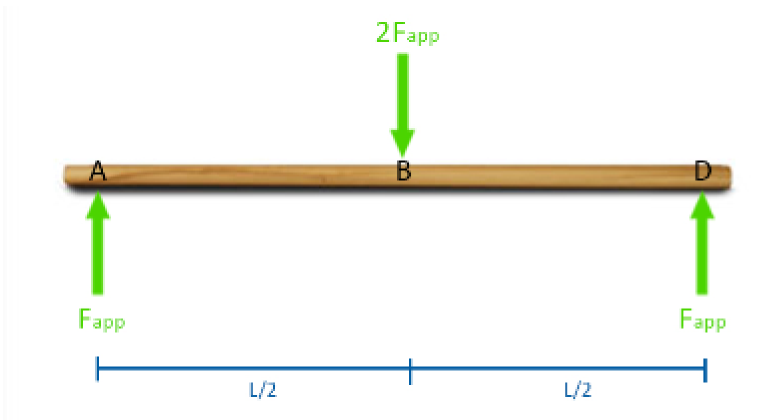


Figure 8. FBD of 3-point bending

The following equation can be used to calculate three point bend, where 0.25FL is the maximum moment, r is the distance from the neutral axis to the edge, and the denominator is second moment of area.

$$\sigma_{max} = \frac{\left(\frac{1}{4}\right) FL(r)}{\left(\frac{\pi}{4}\right) r^4}$$

$$\sigma_{max} = \frac{\left(\frac{1}{4}\right) 2F_{app} 60(1.5875)}{\left(\frac{\pi}{4}\right) 1.5875^4}$$

$$\sigma_{max} = 9.5475F_{app}\ MPa$$

$$117x10^6 = 9.5475F_{app}x10^6$$

$$F_{app} = 12.255N$$

It appears that further reinforcement will be needed based on these calculations.

Four point bending is calculated below.

First, the four forces on the dowel must be calculated. This can be done using the support reactions.

$$F = \sqrt{{B_x}^2 + {B_y}^2}$$

$$F = \sqrt{{6F_{app}}^2 + {F_{app}}^2}$$

$$F = 6.082F_{app}$$

The entire length of the dowel is assumed to be 6 cm (since that is the target width from the constraints), and it is assumed that the forces on either end will act about 1 cm apart from one another. The diagram of this situation can be found below in Figure 9.

The following equation can be used to calculate 4-point bend, where (1/12)FL is the maximum moment, r is the distance from the neutral axis to the edge, and the denominator is the second moment of area.

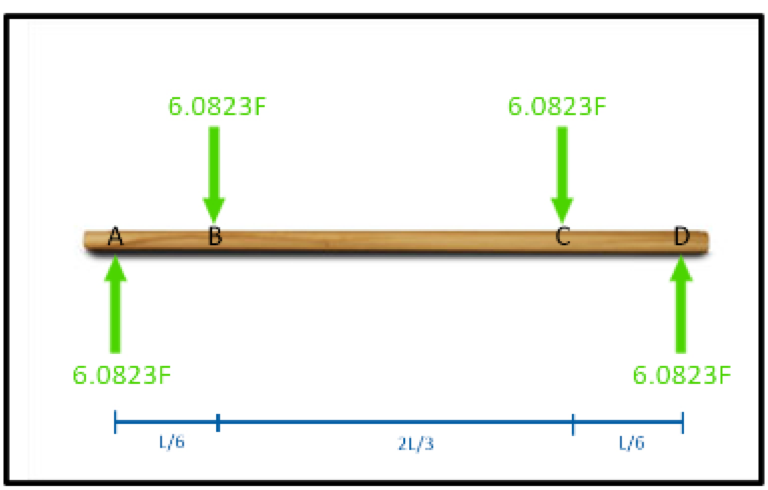


Figure 9. FBD of 4-point bending

$$\sigma_{max} = \frac{-My}{I}$$

$$\sigma_{max} = \frac{\left(\frac{1}{12}\right) FLr}{\left(\frac{\pi}{4}\right) r^4}$$

$$\sigma_{max} = \frac{\left(\frac{1}{12}\right)(2x6.082F_{app})(60)(1.5875)}{\left(\frac{\pi}{4}\right) 1.5875^4}$$

$$\sigma_{max} = 19.095F_{app}\ MPa$$

$$117x10^6 = 19.095F_{app}x10^6$$

$$F_{app} = 6.127N$$

It appears that further reinforcement will be needed for this based on these calculations.

4.2. Design and Member Brainstorming and Optimization

As outlined in Sections 3.1 and 3.2, several considerations were taken when selecting a final design such as a low mass, minimized maximum forces in members, and low compressive forces. While these considerations were essential foundations in the design of a successful truss, optimizing each member individually was another aspect that needed to be addressed. Several ideas to optimize members were brainstormed ahead of any mathematical analyses to address potential issues with the design of compressive and tensile members, plate tear, bearing stress, torsion, and 3-point bending on the weight-bearing dowel. After the mathematical analyses were conducted in the sections above, optimization ideas were narrowed down, and the best solutions were selected for each area of concern.

4.2.1. Design of Compressive and Tensile Members

Over the initial design phase, several solutions were considered preemptively to handle the effects of compressive and tensile members. For compression members, ideas that were considered included box beams, I-beams, square compressive members, and doubling up compression members. First, the design of I-beams was considered. When a beam bends, the top of the beam is typically in compression, and the bottom is in tension as shown in Figure 10 below. These forces are greatest at the very top and very bottom. To make the stiffest beam with the least amount of material, the I-beam is effective as the material is only at the top and bottom sides (known as flanges), connected by a web, as shown below in Figure 10. This works best when the load is parallel to the flange. When the load comes from two directions, it is more effective to use square tubes - or box beams. However, when all the calculations to design the dimensions of the members were done by hand, it was found that box-beams and I-beams would be unnecessarily over-designed. The necessary cross-sectional areas and second moments of inertia were so small that these large compression members would be unnecessary. As it will be shown in the design iterations in the sections to come, it was easier to cut back on mass simply by doubling up compressive members. Another way doubling up compressive members was effective was that it allowed for having square cross-sectional areas. This meant that the second moment of area would be

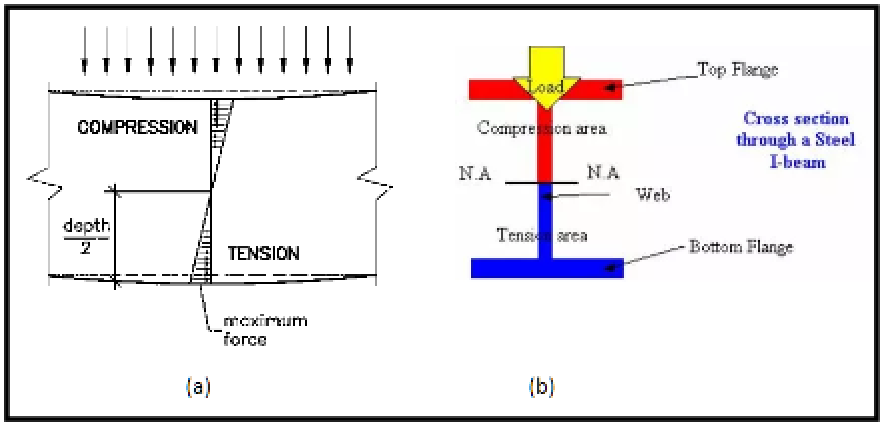


Figure 10. The flange and web design of an I-beam

the same in both axes and would be designed to fail in both directions at the same time.
For tensile members, ideas that were considered included members with a dog bone design. This dog bone design would save mass in the overall truss due to the nature of tensile strength only taking into account the minimal cross-sectional area for tension members. Another consideration for tensile members was plate tearing. Through calculations, an appropriate distance b from the ends of the dog bone members was determined for the pins. A potential problem that was raised with the design of dog bone members was the curvature profile at the ends. As shown in Figure 11, the top member would have a greater concentration of stress at the circled region than the bottom member would. The sharp point creates the stress concentration whereas the gradual change removes this problem. This stress concentration is greater than what is predicted by the calculations; therefore, the design was altered such that the ends were enlarged more gradually. This smooth change removes these stress concentrators, allowing the members to behave more ideally.

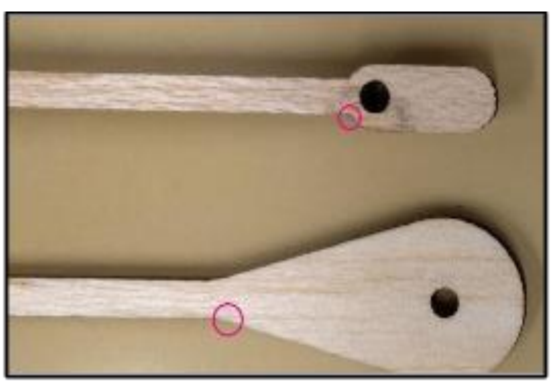


Figure 11. The top member has a greater concentration of stress at the circled region than the bottom member

To account for bearing stress due to the calculations, small circular pieces were glued to the ends of members at the pins (as seen in Figure 12). However, in later iterations, it was determined that like much of the suggestions derived from the calculations, these pieces were unnecessary. Bearing stress was simply not a pressing issue.

Figure 12. An idea to account for bearing stress. Further iterations proved this design decision unnecessary

4.2.2. Dowel Design Considerations

Considerations for dowels also had to be taken throughout the design optimization. Specifically, pin shear and three point bending were areas of concern. To combat pin shear, using multiple dowels at the pins was considered. Using two dowels at a pin would render rotation impossible, and hence, three would have to be used. This was not ideal and it was desired that this be avoided. As a result, pin shear was the limiting factor. The best way to combat this was to use new dowels for the final iteration. To combat the issue of a dowel failing due to 3-point bending (specifically the dowel on which the load was applied), a similar solution of three dowels would have to be used. Alternatively, it was possible to use only one rotating dowel and simply glue another one, half the length of the original dowel, along the top of the rotating dowel. This smaller dowel would not go through the members and hence, not limit rotation.

4.2.3. Torsion of the Truss

Another potential issue was failure due to torsion. Torsion here is not in regards to a single member but in regards to the entire structure as a whole. The truss design that was chosen had room for torsion if the proper precautions were not taken. With the nature of the load being at the free end, it could move the end of the truss to the side if there was not sufficient force opposing this motion. The movement of this load created torsion in the rest of the truss. The truss was not designed to resist this torsion as well as it was designed to resist axial loading (due to two force members). There were a number of ideas on how to reduce this. One was gluing caps to the dowel at either end of a joint. With this, the joint would still rotate freely, but the rotation of the pin holes left and right on the dowel would be restricted by the caps. Another idea was to add cross members in the 3D portion of the truss, between the dowels spanning the space between the two identical trusses. Finally, there was the idea to laser cut the holes smaller, to create a perfect fit. This last idea was not chosen because the balsa wood has enough give that it was unlikely that the perfect fit holes could resist torsion, and would eventually widen. As well, the perfect fit would be very difficult since the laser cutter does not cut perfectly straight vertically (the burning means the cut is actually a little wider at the top versus at the bottom). Forcing the dowels into holes that were too small would pre-stress those holes, which could be problematic in practice. The cross members were not chosen because implementation would be too difficult, the main problem being that it would be difficult to allow the pin joints to rotate. As well, it added a lot of weight. So, in the end caps were chosen because they would theoretically suffice while at the same time being easy to implement and not adding too much extra weight.

4.2.4. Optimizing Available Material

Since the amount of material provided to laser cut all the iterations as well as the final design was a piece of 16x50in. balsa wood, it was important to use the material as efficiently as possible. Through AutoCAD, the members were to be printed as close to one another as possible. Smaller members and bearing circles were to be printed in scrap pieces if necessary. Furthermore, it was decided that to maximize the number of iterations, members and dowels would be reused through multiple iterations if possible. To ensure this, in the first few iterations, the members were not glued to each other, and were rather left loose on the dowels with just the end caps holding them together. This way, after the truss failed, the undamaged members could be removed safely. This reduced the amount of replacement members that needed to be cut for multiple iterations.

4.2.5. Balsa Wood Inconsistencies

One property of the balsa wood that was discovered was that different sheets of balsa wood had different densities, with some sheets varying by as much as a factor of two. This is important because the lower density sheets were far weaker than the higher density sheets. The difference between two pieces of balsa wood is demonstrated below. Figure 13 outlines two identical members under similar bending conditions. The stronger member takes a much larger force to bend (and eventually break) whereas the weaker member bends (and subsequently snaps) under very little force.

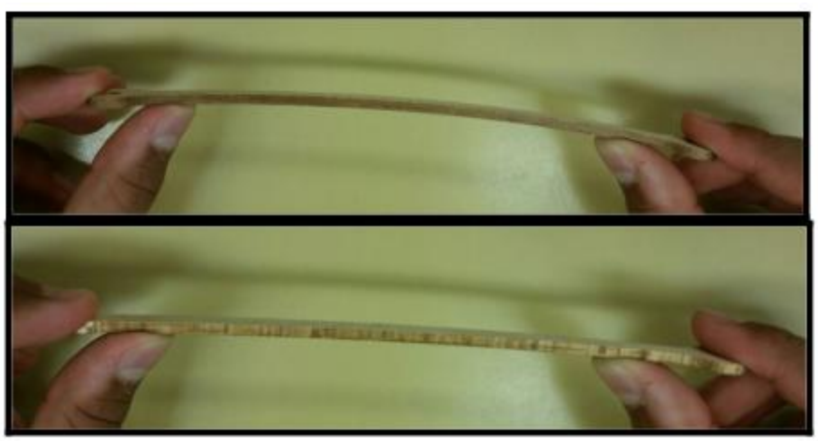


Figure 13. The top image displays the weaker balsa wood while the lower image displays the stronger balsa wood

Figure 14 shows two of the dowel caps, each from a different piece of balsa wood. Each were compressed under a similar force. The shorter one compressed like a sponge while the taller one held firm.

Figure 14. Difference between balsa wood densities in the caps when squeezed with full strength

With regards to the members, it is important to use the stronger members, and to ensure that every member has a similar density. If they don't, the weaker member will break first despite the rest of the members being able to handle much greater forces. The weaker ones would break at much lower forces so it is imperative that they are not used in the design, or else the truss is going to be much weaker than predicted. The caps are also important because they resist torsion. If the truss is trying to rotate and the cap restraining it can compress like a sponge, it won't restrict movement. So, picking caps made of the stronger balsa wood has a very obvious effect on the performance of the truss.

Another important consideration with the balsa wood pertained to the laser cutting orientation. It is advantageous to avoid cutting members against the grain. Balsa wood is an anisotropic material, meaning that the material properties of balsa wood are different along each axis. Laser cutting members parallel to the grain generates much stronger pieces than when they are cut perpendicular to the grain. Cutting perpendicular to the grain yields pieces that are far too weak to use.

4.3. Final Design

A 3D rendering of the final design used in competition can be found in Figure 15 below.

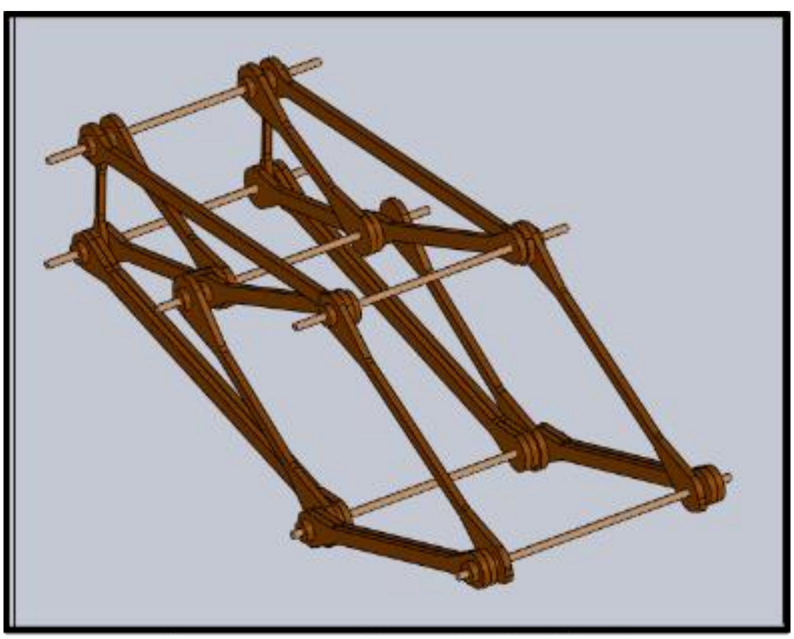


Figure 15. 3D Rendering of the final design

5. Construction and Results

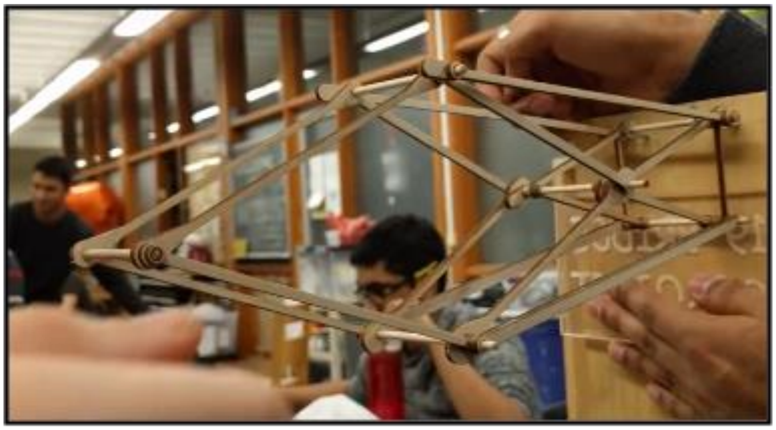


Figure 16. The first design iteration mounted on the test platform

5.1. Design Iterations and Refinements

The first iteration of the truss design was made based on the calculations in the sections above. Every member was created to match the minimal cross sectional areas for every failure mode considered. When this was built and tested, it held approximately 600 grams (its mass was 7 grams), which is far below what the calculations predicted. The truss pre-testing can be seen in Figure 16. This could have been due to a number of factors. These include balsa wood inconsistencies and laser cutter imperfections. In regards to the balsa wood inconsistencies, there is not much that can be done, besides picking pieces that feel stronger. The inconsistency of the balsa wood is difficult to account for in the calculations, and thus would need to be accounted for in the safety factor. With the laser cutter inconsistencies, the laser cutter does not make a perfect cut perpendicular to the cutting surface. This leaves members that have greater widths on one side versus another. This is especially problematic when dealing with very small cross sectional areas, especially those around the pin holes. To combat this problem all of the member's cross sectional areas were increased so that the laser cutter imperfections played less of a role on the truss. This includes both the length of the members and at the pin holes.

The next iteration was laser cut with the new adjustments. Some of the members from the first iteration were re-used since they were not found to be problematic. This saved materials for future iterations. This iteration only held 900 grams (its mass was 8 grams), due to a couple of reasons.

The cross sectional areas had only been increased marginally, and the members that did fail had some areas around the pin joints that were a little small and fragile. To fix this problem, the area around the pins was increased by a considerable amount. The biggest reason that it failed however that was the members all contained stress concentration points that had not been accounted for in the calculations. Sharp corners on members can act as stress concentrators (highlighted in Figure 9 in Section 4.2.1). This property was unknown and thus was not accounted for. Each of these corners took more stress than what would be expected and thus needed to be fixed. To remedy this, each of the corners were smoothed out to create a dog bone shape, removing these stress concentrators. These improvements are seen in Figure 9 in Section 4.2.1 above.

The next iteration was laser cut entirely brand new, since every member had been remedied in some way. There was also one more small addition made. Small caps resembling donuts were printed and glued at either end of the dowels, to ensure than no members fell off (this was a problem when assembling and setting up the previous iterations). This truss weighed 10 grams, but was able to hold 1.3 kilograms (the truss can be found in Figure 17 below).

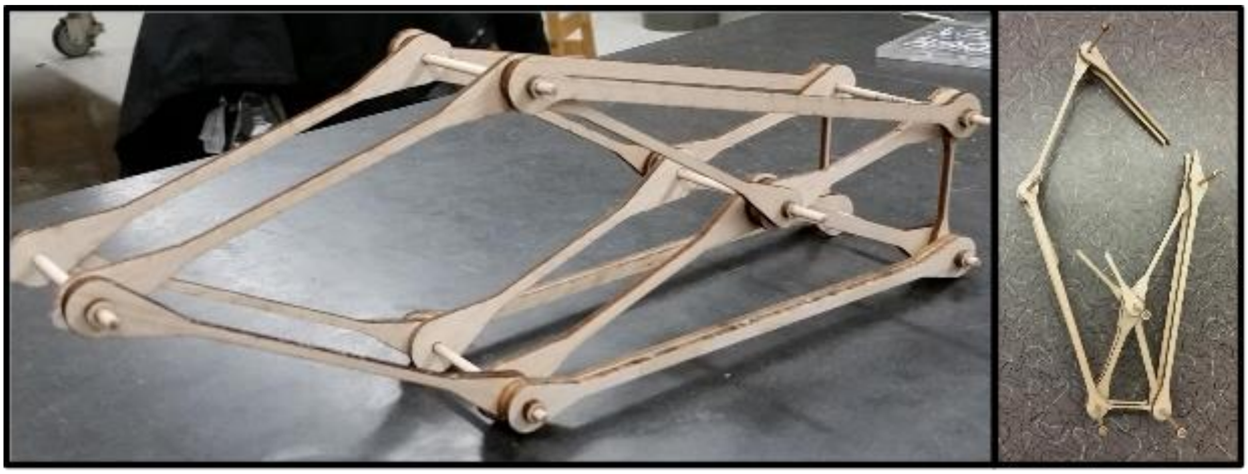


Figure 17. The third truss design before and after testing

The problem with this truss was that it failed due to torsion. The members were able to twist on the dowel pins, and as a result the truss was not very strong at resisting torsion. The weight pulled the truss 'x' to one side while moving the end to the other. This created a twisting problem which caused multiple members to fracture at once. The members that did fracture broke with the characteristic 45 degree angle seen in typical torsional failure. This is seen below in Figure 18. Even though PV was met, it was desirable to fix this problem since there was still time and material remaining. For the next iteration, to restrict the torsion, dowel caps were glued on both sides, instead of just one. These caps were glued to the dowels as tight to the members as possible, in the attempt to create a sort of vice between the caps, pinching the members in between. The members were still free to rotate. In addition to these changes, the members that broke in torsion were additionally fortified, in case torsion was still an issue, they would last longer.

Figure 18. The broken member with characteristic 45 degree angle seen in torsional failure

This edition was cut and assembled. The truss pre-test can be found below in Figure 19. When measured it had a mass of 15 grams and held 3 kilograms. The failure problem this time was that the dowel at the hooks bent and broke. As well, the two long tension members attached to it also broke. With regards to the dowel that broke, it was a recycled dowel that had been used and glued before. This may have previously weakened it, so new dowels would be used in the final iteration. With regards to the tension members, the failure in this case was plate shear, so the ends of the members were lengthened to increase the ‘b’ value to combat plate shear. Since there was only room for one more cut, these were the only changes made, and they could not be tested.

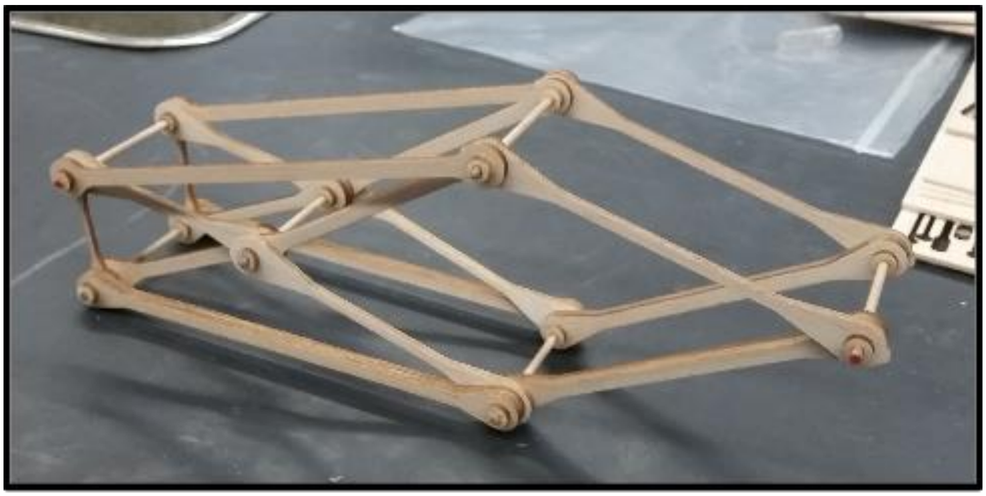


Figure 19. The fourth design iteration before testing

The final design was then cut, and assembly began. However, during assembly it was found that the new members cut did not have the same properties as the ones used in previous tests. They were lighter and seemed weaker. The new members cut were compared with the ones from the truss that failed at 3 kilograms and these suspicions were confirmed. The density was not even close to the same between the two. The new members would snap under very little force compared to the members from the last test that had been used. This was problematic as these members were far weaker than what had been tested with, and subsequently would break under much smaller forces. It would be a gamble to assume that the reduction in mass (since this balsa wood was not as dense) would offset the reduction in mass held. To try and fix this problem before competition, all the members from previous trials and the new printed ones were compared. They were sorted by which part of the truss they belonged to and the strongest two for every member were chosen. They were each meticulously tested using a rudimentary bending test to try and find the strongest members. Since many of the members used in the final were already used in the previous test, there was concern that they may have issues from already being loaded before. As well, many of the pin holes had been stressed enough to make them loose, which made torsion a potential problem again. As a result, the caps had to be glued perfect to pinch the members together to prevent rotation, while still ensuring that they were all pin joints. The truss was very carefully and meticulously assembled, trying to ensure that everything was assembled perfectly. After very careful gluing, the final truss was ready for competition day. The fact that new members could not be used for the entire truss left some concerns, but there was also optimism that individually picked members may actually improve performance. Balsa wood has a very high standard deviation for its density, and it is difficult to tell which members may be problematic. It only takes one member that is a little weaker than designed to fail to cause the whole truss to fail. It is very possible that this was an issue in previous tests. It was hoped that hand-picking each member individually may have solved that problem, reducing the role that varying densities could play. The final truss had a mass of just over 16 grams. It can be seen in Figure 20 below.

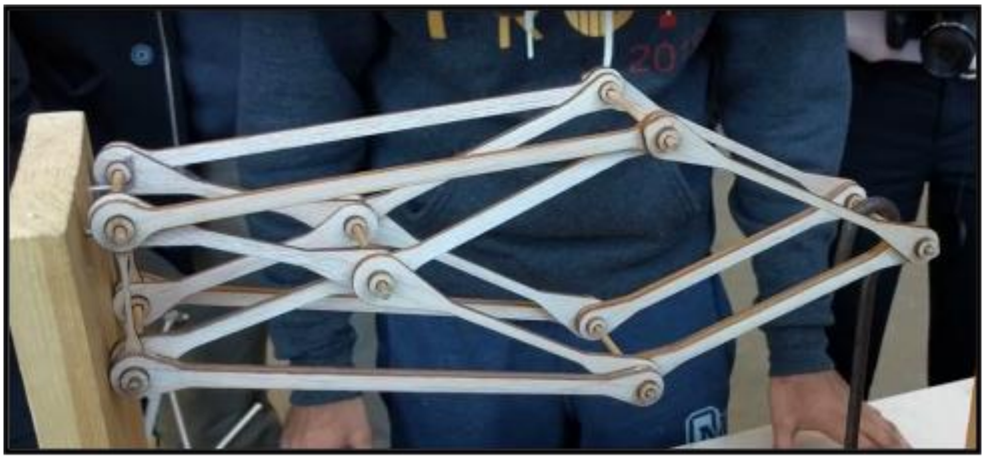


Figure 20. The final truss design on the test platform

5.2. Final Design Results

Come competition day, the official mass of the truss was about 16.08 grams. This meant to reach the desired 125 performance value, the truss would need to hold 2.5 kilograms. One idea that was had was to set the truss on the hooks so that the hooks contacted the dowel caps. It was done to make sure that the hooks were as close to the members as possible. Upon initial loading, it appeared as though the truss was moving ever so slightly to one side. To counteract this, the weight was moved slightly off center to keep the truss aligned. The truss held up until 2.5 kilograms, where it was observed that the off-center load was starting to bend the truss back the other way. The load was re-centered, and further loading continued. In the end, the truss held 5 kilograms, before breaking at 5.5 kilograms. The broken truss can be found in Figure 21 below.

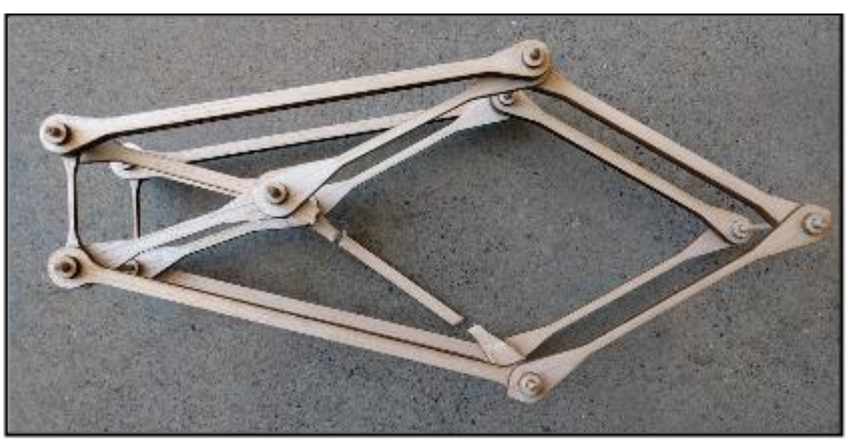


Figure 21. The final design after breaking. The broken member is displayed to illustrate break location.

The new dowel and the hook placements likely helped because in the end the failure mode was not at the dowel, but rather at two of the inner compression members. Since these members had never been problematic they had not been considered as closely during optimization as the other members, which is most likely what lead to their failure. Regardless, the truss most likely would have failed when 6 kilograms was loaded anyway because the dowel at the hooks was starting to bend when the inner compression members failed.

The performance value was calculated and found to be 311. This was an enormous success as the hard work and time put in, both designing and optimizing, and reiterating paid off. This score was good for first in the class, and the only design to eclipse the 300 PV mark. This truss performed phenomenally well; it was exciting to see the success it had.

It is difficult to compare the math behind the truss to the final product since the final product changed so much from the initial calculations. In the end, despite more than doubling the mass of the truss, the truss still did not even hold half of what it was predicted to hold. This can most likely be chalked up to a number of reasons. These reasons include the inconsistency of the balsa wood, the imperfections of the laser cutter, the stress concentrators, as well as other modes of failure that were not taken into consideration because they are unknown. The fact is that there are most likely other modes of failure, and other properties that could not be factored in because they are unknown. The math assumes ideal, perfect conditions and unfortunately, those do not exist in real-world applications.

6. Recommendations and Conclusions

6.1. Next Steps

In the future there are few things that could be done differently to potentially yield better results. The main thing would be to conduct more test iterations. Time and supplies limited the number of tests that could be done, however if there was more time and material available, the truss could have been improved further. Another recommendation would be consistent balsa wood selection. There were difficulties with the final truss because the balsa wood that was left to cut with was much weaker than the balsa wood used in the tests. Ensuring that none of the weaker balsa wood is used would go a long way in strengthening this truss. Also, if more material were available, members would not need for be reused for subsequent iterations. This is prevent weakening due to strain. This weak balsa wood meant that members used in previous tests that had not broken had to be used for the final competition truss. This luckily was not a huge factor but may have played a role considering that the members and pin holes were pre-stressed. Furthermore, while fairly insignificant, it would save weight to trim the dowels closer to the pins. There was a lot of excess dowel weight extending past the dowel caps. This was unnecessary weight that could have been removed. Strengthening the truss could easily be done through strengthening the compression members that broke (the ones that failed in the competition) and by increasing the number of dowels in the joint at the hooks. If the compression members had not failed, the dowel at the joints would have. Using multiple dowels would have strengthened this, allowing it to hold more mass. Finally, a more complete failure mode analysis could be done. This analysis was one using all modes of failure learned in class. If there was more time, other failure modes and material properties could have been researched to potentially close the gap between the calculation predictions and the actual test findings. These suggestions could have increased the performance of the truss, however the truss produced, based on the knowledge, time, and materials available performed admirably.

6.2. Method of Execution

The final truss iteration weighed 16.08 grams and was able to hold the weight of 5.0kg, yielding a performance value of around 311. The design process consisted of four predominant steps including preliminary design considerations, a complete mathematical analysis, optimization of members, and the construction of five iterations of the truss with calculated refinements in each iteration. Preliminary design considerations included creating designs with a low mass, with members that had low maximum forces and low compressive forces. When a design was selected, a full mathematical analysis was performed to calculate the forces in each member as well as consider each mode of failure discussed in the MTE 219 course. Then, each compressive and tensile member was optimized to address these modes of failure, and extra precautions were taken to account for torsion and the anisotropic nature of balsa wood. Through various construction iterations of the truss, observing how and where each iteration failed allowed the next version to be better fortified through increasing the size of the dog bone members, adding bearing circles at the ends to prevent torsion, and increasing the distance between the pins and ends of the members to prevent plate tearing. Ultimately, the refinement process led to the final and fifth iteration of the truss, which on the day of the competition, was able to support a winning PV of 311.

6.3. Real World Application

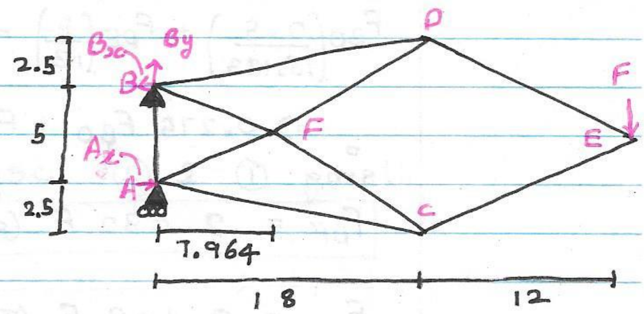
This project was integral in the professional and academic development of each individual that participated. It allowed each team to discover the frustration involved, attention to detail necessary, and arduous effort required to undertake any design project, whether in the setting of a university, or as professional engineers in the real world. More importantly, it brought to light the amount of effort and careful attention to detail which is required when the project at hand directly affects society and other individuals. As professional engineers designing a bridge, programming a rocket, or building an MRI machine, all factors need to be considered. When the project at hand will be used by other beings, human or otherwise, there is no room for error, no excuses of how “the math lied” and no justification for “this was never taught.” Each and every factor must be carefully considered, mathematically analyzed, rigorously optimized and refined. The product at hand must be constructed with the utmost attention to quality and workmanship, whether it be clean code or good quality materials. All of this must be done staying within the criteria and constraints, while knowing which of the criteria to accept the expense of others.

This duty to society as engineers is evident in the Iron Ring received by each graduate. While a correlation between the Iron Ring and the failure of the Quebec Bridge is debatable, the Quebec Bridge stands as an excellent example to this very duty. The bridge failed twice at the cost of 88 lives. The first failure was attributed to miscalculations that were never rigorously challenged, and the next was due to the use of poor materials as an attempt to stay within budget. Regardless, these compromises speak volumes when compared to the non-critical nature of this particular project. This project allows each individual to step back and appreciate wholeheartedly the hard work and beauty of every engineering feat, whether it be a wooden structure in the park, or the world's tallest building.

7. References

[1] Autodesk, "Autodesk ForceEffect(tm),". [Online]. Available: https://forceeffect.autodesk.com/frontend/fe.html. Accessed: Mar. 21, 2016.

FORCE ANALYSIS OF 2D TRUSS



For complete truss,

$\Sigma F_y = 0$

$\Rightarrow A_y = 0$ [$\because$ only horizontal support reaction]

$\boxed{B_y = F}$

$\Sigma F_x = 0$

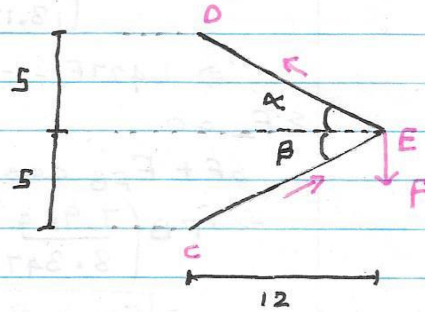
$\Rightarrow B_x = A_x$

$\Sigma M_B = 0 \quad \Rightarrow \quad 5 B_x = 30F$

$\Rightarrow \boxed{B_x = 6F}$

$\boxed{\Rightarrow A_x = 6F}$

- At joint E,



$\Sigma F_x = 0$

$\Rightarrow F_{ED} \cos\alpha = F_{EC} \cos\beta$

$\cos\alpha = \cos\beta$ [$\because \alpha = \beta$]

$\Rightarrow F_{ED} = F_{EC} = F_1$

$\Sigma F_y = 0$

$\Rightarrow F = F_{ED} \sin\alpha + F_{EC} \sin\beta$

$= 2F_1 \sin\alpha$

$= 2F_1 \, 5/13$

$\Rightarrow \boxed{F_{ED} = 1.3F \text{ (T)}}$

$\boxed{F_{EC} = 1.3F \text{ (C)}}$

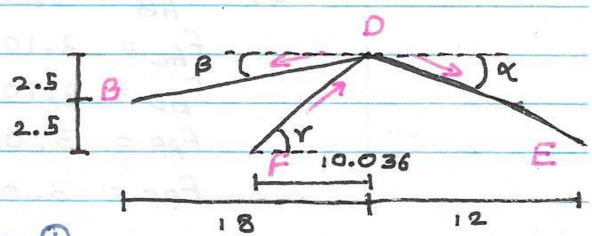
- At joint D,

$\Sigma F_x = 0$

$F_{DE} \cos\alpha + F_{DF} \cos\gamma = F_{BD} \cos\beta$

$\Rightarrow 1.3F \left(\frac{12}{13}\right) + F_{DF} \left(\frac{10.036}{11.213}\right) = F_{BD} \left(\frac{18}{18.173}\right)$

$\Rightarrow 1.2F + (0.895) F_{DF} = (0.99) F_{BD}$ — (1)



$\Sigma F_y = 0$

$F_{BD} \sin\beta + F_{DE} \sin\alpha = F_{DF} \sin\gamma$

$F_{BD}\left(\frac{2.5}{18.173}\right) + F_{DE}\left(\frac{5}{13}\right) = F_{DF}\left(\frac{5}{11.213}\right)$

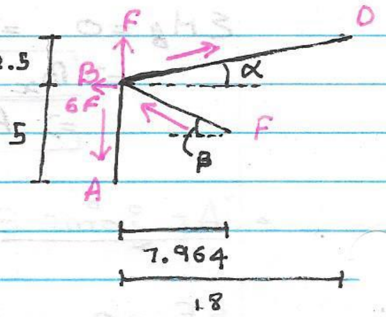
$\Rightarrow 0.275\, F_{BD} + F = F_{DF}(0.891)$ — (2)

Using (1) & (2) we get,

$\boxed{F_{DF} = 2.082\, F\ (C)}$

$\boxed{F_{BD} = 3.109\, F\ (T)}$

- At joint B,



$\Sigma F_y = 0$

$\Rightarrow F + F_{BD} \sin\alpha + F_{FB} \sin\beta = F_{AB}$

$F + F_{BD}\left(\frac{2.5}{18.173}\right) + F_{FB}\left(\frac{2.5}{8.347}\right) = F_{AB}$

$\Rightarrow 1.427F = -0.299\, F_{FB} + F_{AB}$ — (3)

$\Sigma F_x = 0$

$6F + F_{FB} \cos\beta = F_{BD} \cos\alpha$

$\Rightarrow F_{FB}\left(\frac{7.964}{8.347}\right) = \left(\frac{3.109 \times 18}{18.173}\right) F - 6F$

$\Rightarrow \boxed{F_{FB} = 3.061\, F\ (T)}$

$\Rightarrow \boxed{F_{AB} = 0.511\, F\ (T)}$

Due to symmetry about the x axis, members on the lower half have forces of the same magnitude by nature of forces are opposite

$\Rightarrow$

$F_{AB} = 0.511F\ (T)$	$F_{CF} = 2.082F\ (T)$
$F_{AC} = 3.109F\ (C)$	$F_{DF} = 2.082F\ (C)$
$F_{BD} = 3.109F\ (T)$	$F_{CE} = 1.3F\ (C)$
$F_{AF} = 3.061F\ (C)$	$F_{DE} = 1.3F\ (T)$
$F_{BF} = 3.061F\ (T)$	

Plate Rupture (Tension)

$\sigma = \frac{F}{A}$ $\sigma_{max} = 14.6\,MPa$ $1mm^2 = 1E\text{-}6 m^2$ $W = width$

AB $\rightarrow \sigma = \frac{F}{A}$

$14.6E6 = \frac{0.5F}{A}$

$A = \frac{0.5F}{14.6E6}\ m^2$

$A = \frac{0.5F}{14.6}\ mm^2$

$td = \frac{0.5F}{14.6}\ mm^2$

BD $\rightarrow \sigma = \frac{F}{A}$

$14.6E6 = \frac{3.0848F}{A}$

$A = \frac{3.0848F}{14.6}\ mm^2$

$tw = \frac{3.0848F}{14.6}\ mm^2$

BF $\rightarrow 14.6E6 = \frac{3.0862F}{A}$

$A = \frac{3.0862F}{14.6}\ mm^2$

$tw = \frac{3.0862F}{14.6}\ mm^2$

CF $\rightarrow 14.6E6 = \frac{2.0730F}{A}$

$A = \frac{2.0730F}{14.6}$

$tw = \frac{2.0730F}{14.6}$

DE $\rightarrow 14.6E6 = \frac{1.3000F}{A}$

$A = \frac{1.3000F}{14.6}\ mm^2 \rightarrow tw = \frac{1.3F}{14.6}\ mm^2$

Pin Shear

Max Force at AC/BD = 3.0862F

$\tau = \frac{F}{A} = \frac{F}{\frac{\pi}{4}(d)^2}$ $\tau_{max} = 23\,MPa$

$d = 0.125' = 3.175\,mm$

$23E6 = \frac{3.0862F}{\frac{\pi}{4}d^2}$

$(3.175E\text{-}3)^2 = \frac{3.0862F}{23E6\ \frac{\pi}{4}}$

$F = 59.004\ N$

Plate Shear - Tensile Members only

$\tau = \dfrac{F}{2bt}$ $\qquad \tau_{max} = 2\cancel{E6}$ MPa $\qquad 1mm^2 = 1E\text{-}6\,m^2$

AB $\rightarrow \tau = \dfrac{F}{2bt}$

$2E6 = \dfrac{0.5000F}{2 \cdot bt}$

$bt = \dfrac{0.5F}{4}\ mm^2$

$bt = 0.125F\ mm^2$

BD $\rightarrow \tau = \dfrac{F}{2bt}$

$2E6 = \dfrac{3.0848F}{2bt}$

$bt = \dfrac{3.0848F}{4}$

$bt = 0.7712F\ mm^2$

BF $\rightarrow 2E6 = \dfrac{3.0862F}{2bt}$

$bt = \dfrac{3.0862F}{4}$

$bt = 0.77155F\ mm^2$

CF $\rightarrow 2E6 = \dfrac{2.0730F}{2bt}$

$bt = \dfrac{2.0730F}{4}\ mm^2$

$bt = 0.51825F\ mm^2$

DE $\rightarrow 2E6 = \dfrac{1.3000F}{2bt}$

$bt = \dfrac{1.3F}{4}\ mm^2$

$bt = 0.325F\ mm^2$

Bearing Stress - Pick minimum of pin normal and plate normal → plate normal

$\sigma_{max} = 14.6 MPa$

AB → $\sigma = \frac{F}{td}$

$14.6E6 = \frac{0.5F}{td}$

$td = \frac{0.5}{14.6} F \; mm^2$

AC → $\sigma = \frac{F}{td}$

$14.6E6 = \frac{3.0848F}{td}$

$td = \frac{3.0848F}{14.6} \; mm^2$

BD → $14.6E6 = \frac{3.0848F}{td}$

$td = \frac{3.0848F}{14.6} \; mm^2$

AF → $14.6E6 = \frac{3.0862F}{td}$

$td = \frac{3.0862F}{14.6} \; mm^2$

BF → $14.6E6 = \frac{3.0862F}{td}$

$td = \frac{3.0862F}{14.6} \; mm^2$

DF → $14.6E6 = \frac{2.0730F}{td}$

$td = \frac{2.0730F}{14.6} \; mm^2$

CF → $14.6E6 = \frac{2.0730F}{td}$

$td = \frac{2.0730F}{14.6} \; mm^2$

CE → $14.6E6 = \frac{1.3F}{td}$

$td = \frac{1.3F}{14.6} \; mm^2$

DE → $14.6E6 = \frac{1.3F}{td}$

$td = \frac{1.3F}{14.6} \; mm^2$

☆ $td = 3.175^2$ if we don't major mod so, find min F for all these, which is when $F = 3.0862F$

$3.175^2 = \frac{3.0862F}{14.6}$

$F = 47.69 \; N$

Compressive Normal - will fail at buckling unless F gets really high, and buckling members are excellent

$$F_{cr} = \frac{4\pi^2 EI}{l^2}$$ → 4 accounts for members fixed at both ends

AC → $3.0848F = \frac{4\pi^2 \cdot 3.66E9 \cdot I}{0.18173^2}$ N, GPa, m

$I = 6.46775E\text{-}13 F\,m^4$

$I = 6.46775E\text{-}1 F\,mm^4$

$1m^4 = (10^3)^4\,mm^4$

$1m^4 = 10^{12}\,mm^4$

AF → $3.0862\,F = \frac{4\pi^2 \cdot 3.66E9 \cdot I}{0.08347^2}$

$I = 1.48814E\text{-}13\,F\,m^4$

$I = 1.48814E\text{-}1\,F\,mm^4$

DF → $2.0730\,F = \frac{4\pi^2 \cdot 3.66E9 \cdot I}{0.11213^2}$

$I = 1.8038571E\text{-}13\,F\,m^4$

$I = 1.8038571E\text{-}1\,F\,mm^4$

CE → $1.3000F = \frac{4\pi^2 \cdot 3.66E9 \cdot I}{0.13^2}$

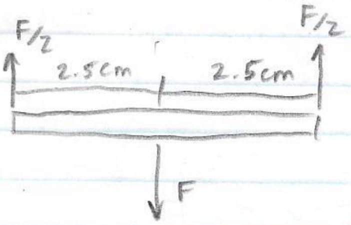
$I = 1.52051E\text{-}13\,F\,m^4$

$I = 1.52051E\text{-}1\,F\,mm^4$

Square block - $I = \frac{t^4}{12}$ → for AC we get $t \cong 2.5$ mm .... (F=60)

for CE we get $t \cong 1.74$ mm

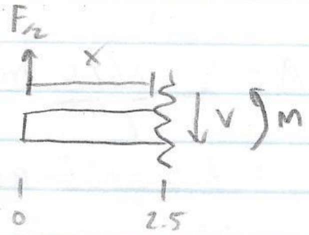
3-Point Bend for weight hang end



F_s $\sigma_{R fail}$ (given in docs)

$$\sigma = \frac{FL}{\pi R^3}$$

$0 < x < 2.5$



$M = \frac{1}{2} F x$

M_{max} at center $= \frac{1}{2} \cdot F \cdot 25mm$

$M_{max} = 12.5F \ N \cdot mm$

$$\sigma_{max} = \frac{Mr}{I} = \frac{Mr}{\frac{\pi}{4} r^4} = \frac{4M}{\pi r^3} = \frac{4 \cdot 12.5F}{\pi r^3} = 0.4973F$$

$r = 3.175mm$
all lengths in mm
F in N

$$\frac{50F}{\pi r^3} = \frac{FL}{\pi r^3}$$

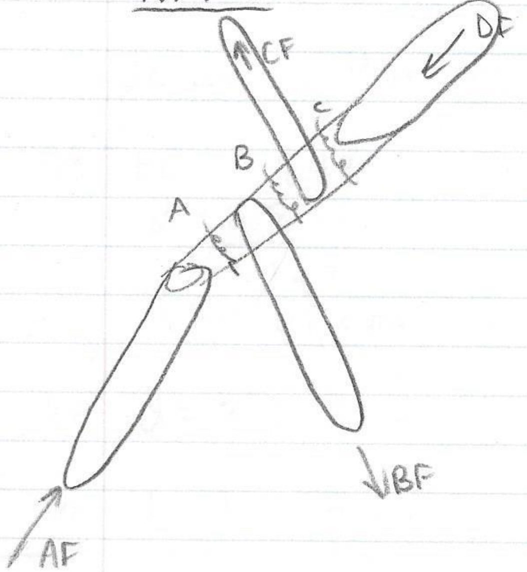
$L = 50mm$

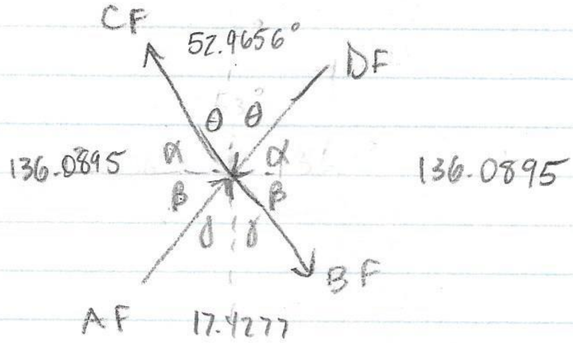
want relationship b/w r at center and F....

$\boxed{\text{Disregard This}}$

OK ✓

Point F





$\theta = 26.4828$
$\alpha = 63.5172$
$\beta = 72.5723$
$\gamma = 8.71385$

$F_A = F_{AF} = 3.0862F$

$$23 = \frac{3.0862F}{\frac{\pi}{4}(3.125)^2}$$

$F = 57.16\ N$

$F_B = F_{AF} + F_{BF}$

$F_B = 3.0862F \sin 8.71385\, i + \cancel{3.0862F \cos 8.71385}\, j + 3.0862F \sin 8.71385\, i$
$\quad - 3.0862F\, \cancel{\cos 8.71385}\, i$

$F_B = 6.1724 \sin 8.71385\, F$

$F_B = 0.76703\, F$

$F_C = 0.76703F\,\hat{i} - 2.0730F \cos 63.5172\,\hat{i} + 2.0730F \sin 63.5172\,\hat{j}$
$\quad -0.1574F\,\hat{i} + 1.85547\,j$

Solving for Cross-Sectional Areas

Assume 5.5kg = 54N

Tension Members

AB $td = 1.849$
$d = \frac{1.849}{3.175}$
$d = 0.582$ mm

BD $td = \underline{11.40953}$
$d = \frac{11.40953}{3.175}$
$d = 3.5935$ mm

BF $d = 3.595$ mm

CF $d = 2.4149$

DE $d = 1.5144$ mm

AB $b = 2.126$ mm

BD $b = 13.116$ mm

BF $b = 13.122$ mm

CF $b = 8.814$ mm

DE $b = 5.528$ mm

AB $t = 0.582$ mm

BD $t = 3.594$ mm

BF $t = 3.595$ mm

CF $t = 2.415$ mm

DE $t = 1.514$ mm

Bearing thick
AB

AB = $0.582 \cdot 50 \cdot 3.175 + 0.582 \cdot 2.126 \cdot 3.175 = 96.32mm^3$
BF = $3.595 \cdot 83.47 \cdot 3.175 + 3.595 \cdot 13.122 \cdot 3.175 = 1102.513mm^3$
DE = $1.5144 \cdot 130 \cdot 3.175 + 1.5144 \cdot 5.528 \cdot 3.175 = \underline{651.648}mm^3$
BD = $3.5935 \cdot 180.173 \cdot 3.175 + 3.5935 \cdot 13.116 \cdot 3.175 = 2205.3\,mm^3$
CF = $2.4149 \cdot 112.13 \cdot 3.175 + 2.4149 \cdot 8.814 \cdot 3.175 = 927.3mm^3$

4983-0958

116.8765

Compression

AC $t = 3.594$ mm DF $t = 2.415$ mm

AF $t = 3.595$ mm CE $t = 1.514$ mm

AC $t = 4.525$ mm DF $t = 3.288$

AF $t = 3.134$ mm CE $t = 3.151$ mm

AC = 3.9258 cm^3
AF = 0.9181 cm^3 → 1.~~5~~
DF = 1.320 cm^3
CE = 1.39 cm^3

Bearing
Make all 1 member

Plate Tear $\tau = \frac{F}{2bt}$

max $\tau = 2.05$ MPa

$2.05E6 = \frac{76.531}{2bt_{BC}}$

$bt_{BC} = 18.666 \text{ mm}^2$

$2.05E6 = \frac{32.5}{2bt_{CE}}$

$bt_{CE} = 7.926 \text{ mm}^2$

$2.05E6 = \frac{51.948}{2bt_{DF}}$

$bt_{DF} = 12.67 \text{ mm}^2$

$2.05E6 = \frac{77.33}{2bt_{BF}}$

$bt_{BF} = 18.861 \text{ mm}^2$

$2.05E6 = \frac{12.352}{2bt_{AB}}$

$bt_{AB} = 3.013 \text{ mm}^2$

$14E6 = \frac{77.195}{td_{AD}}$

$= 5.514$

$14E6 = \frac{51.213}{td_{CF}}$

$td_{CF} = 3.658 \text{ mm}^2$

Bearing Stress $\sigma = \frac{F}{td}$

$\sigma_{plate} = 14$ MPa

$\sigma_{pin} = 117$ MPa

$14E6 = \frac{76.531}{td_{BC}}$

$td_{BC} = 5.467 \text{ mm}^2$

$14E6 = \frac{32.5}{td_{CE}}$

$td_{CE} = 2.321 \text{ mm}^2$

$14E6 = \frac{51.948}{td_{DF}}$

$td_{DF} = 3.711 \text{ mm}^2$

$14E6 = \frac{77.33}{td_{BF}}$

$td_{BF} = 5.236 \text{ mm}^2$

$14E6 = \frac{12.352}{td_{AB}}$

$td_{AB} = 0.882 \text{ mm}^2$

$14E6 = \frac{77.043}{td_{AF}}$

$td_{AF} = 5.503 \text{ mm}^2$

$14E6 = \frac{32.500}{td_{ED}}$

$td_{ED} = 2.321 \text{ mm}^2$

Group 28: Azisur Venkatesan, Sudharsan;Bhutta, Umar Zahoor;Smith, Ryan Michael

APPENDIX B: MTE219 PROJECT REPORT EVALUATION

Category	Criteria	Grade	
Format, Appearance, Writing Style	Clarity of communication Spelling, grammar and punctuation Organization and structure Neatness and completeness Title page, table of contents, lists of figures and tables Print quality and binding	9-10 7-8 5-6 0-4	Excellent Good Marginal Unsatisfactory **10** /10
Executive summary	Purpose Methodology Results	3 2 1 0	Excellent Good Marginal Missing 3 /3
Design: Description	Review of the design problem Design constraints and criteria Materials and properties: report your lab results (data and analysis) here *-2 (Compare Lab Results with provided properties)*	15-17 12-14 8-11 0-7	Excellent Good Marginal Unsatisfactory **15** /17
Design: Synthesis	Three conceptually different designs Free hand sketches and features review Concepts evaluations Design selection *-2 (Quantitative selection criteria)*	13-15 10-13 7-10 0-7	Excellent Good Marginal Unsatisfactory **13** /15
Design: Analysis	Completeness of the analysis *+2 (Explicit soultion of member geometry as a function of applied load)* Adequacy and suitability of analysis Accurate and free of errors analysis; Analysis accuracy validation *+1 (Material Anisotropy)* *+1 (Torsional Stability optimization)* Design optimization *+1 (Stress concentration optimization)* Drawings of the final design	27-30 21-26 15-20 0-14	Excellent Good Marginal Unsatisfactory **35** /30
Design: Construction & Testing	Test results and analysis Design refinements	9-10 7-8 5-6 0-4	Excellent Good Marginal Unsatisfactory **10** /10
Project Overall	Creativity *+3 (Design 3 Creativity)* Lab activity (analysis and presentation) Organization and construction execution	13-15 10-13 7-10 0-7	Excellent Good Marginal Unsatisfactory **18** /15

General Comments: It is evident the amount of work that was put into your report which excellently documented the design and analysis process. Your observations on the effects of stress concentrations, material anisotropy, torsional stability etc. are by far the most comprehensive explanations which were presented that explain the differences between the analytical prediction and the actual result. Your real world application section also displays the mark of thinking like a true engineer. Your explicit solution of the problem analytically was very well laid out and made your design iteration and subsequent calculations very quick as you converged to a further optimized physical result with a better correlating analytical model. Excellent job, all of your group should be very proud of this great achievement.

Total **104** /100